Possibility of using dual frequency to control chaotic oscillations of a spherical bubble

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Abstract

Acoustic cavitation bubbles are known to exhibit highly nonlinear and unpredictable chaotic dynamics. Their inevitable role in applications like sonoluminescence, sonochemistry and medical procedures suggests that their dynamics be controlled. Reducing chaotic oscillations could be the first step in controlling the bubble dynamics by increasing the predictability of the bubble response to an applied acoustic field. One way to achieve this concept is to perturb the acoustic forcing. Recently, due to the improvements associated with using dual frequency sources, this method has been the subject of many studies which have proved its applicability and advantages. Due to this reason, in this paper, the oscillations of a spherical bubble driven by a dual frequency source, were studied and compared to the ones driven by a single source. Results indicated that using dual frequency had a strong impact on reducing the chaotic oscillations to regular ones. The governing parameters influencing its dynamics are the secondary frequency and its phase difference with the fundamental frequency. Also using dual frequency forcing may arm us by the possibility of generating oscillations of desired amplitudes. To our knowledge the investigation of the ability of using a dual frequency forcing to control chaotic oscillations are presented for the first time in this paper.

1 Introduction

Cavitation bubbles, driven in motion by an acoustic field, are the example of highly nonlinear forced oscillators. The study of their behavior, recently, has attracted lots of attention and it is shown that this phenomenon exhibit highly nonlinear and complex dynamics both experimentally [1, 2, 3] and numerically [4, 5, 6, 7]. Besides their complex behavior, acoustically driven bubbles have lots of advantageous applications in material science, sonoluminescence and sonochemistry [8, 9, 10, 11, 12], sonofusion [13] and medical procedures such as: lithotripsy, diagnostic imaging, drug and gene delivery, increasing membrane permeability, opening blood brain barrier and high intensity focused ultrasound (HIFU) surgery [14, 15, 16, 17, 18]. In all of these applications a conventional single frequency acoustic source has been used to provide the bubble oscillations. As an alternative to this conventional approach recently using dual frequency sources has lured a great amount of attention in order to improve the outcome of the applications involving acoustic cavitation. Dual frequency has been used to enhance sonoluminescence phenomenon [19, 20, 21, 22, 23] resulting in an increase in the light emission up to 300 percent. In ultrasound imaging, it introduces a robust back scatter of ultrasound agents even at low intensity frequencies, which can be optimized up to 200 percent brighter as compared to single frequency employment [24]. In sonochemistry, using two harmonics has shown to increase the energy efficacy and enhance sonochemical reactors function [25, 26, 27, 28]. In therapeutic applications such as sonodynamic therapy and HIFU, using a dual frequency source is shown to greatly increase the treatment efficacy [29, 30, 31, 32]. In the above mentioned applications, an optimum employment of bubbles suggests that chaotic oscillations be reduced, because when the bubble motion gets chaotic, its behavior becomes unpredictable and very hard to control. Therefor, reducing chaotic oscillations can be the first step in precisely controlling the bubble dynamics and can provide more beneficial outcomes.

In applications involving cavitation, the parameters like the viscosity, surface tension or the diameter of the bubble are determined by the media and type of the application. Therefore, in order to carry out control strategies in the system, the only remaining parameter that can be dealt with, is the perturbation of the forcing term. As mentioned using dual frequency sources is practical and has been proved to have many advantages in an increasing number of applications. For this reason, in order to stabilize the bubble motion,

we perturb the bubble single frequency forcing by introducing a secondary frequency drive to the system. In this respect firstly, through applying the methods of chaos physics, nonlinear dynamics of a spherical cavitation bubble driven by a single frequency source was studied in this paper. The considered control parameters were the applied pressure, frequency and the bubble initial radius. An evaluation upon the stabilizing effect of a dual frequency source on chaotic oscillations was carried out. For a fixed initial frequency, different values of the secondary frequency along with its phase difference with the fundamental were studied. These were done through plotting and analyzing the bifurcation diagrams and the lyapunov exponent spectra. The reason for using these analysis is that in the absence of any direct and reliable mathematical methods, bifurcation diagrams and largest lyapunov exponent spectra are very useful by providing a precise view on the dynamical behavior of the system over a wide range of control parameters. They can be used as powerful tools to determine suitable values for the secondary frequency and the phase difference. For this reason we analyzed the consequences before and after applying the introduced method through plotting the bifurcation diagrams, lyapunov exponent spectra and time series of the bubble.
Results were promising as they indicated an accessible stabilizing role of a dual frequency source on chaotic oscillations. The effective control parameters were found to be the value of the secondary frequency and the phase difference.

2 The Bubble model

The bubble model used for the numerical simulation is derived in [4] and is a modification of the bubble model formulated by Prosperetti[33] from Keller-Miksis bubble equation[34]. The reason for using this model is that it generated results in good agreement with those obtained by solving the full partial differential equations of fluid dynamics and is suitable for a wide range of amplitudes of oscillations [35]. It is is given by (1):

$$\begin{aligned}(1-\frac{\dot{R}}{c})R\ddot{R}+\frac{3}{2}\dot{R}^2(1-\frac{\dot{R}}{3c}) &= (\frac{P}{\rho})+\frac{1}{\rho c}\frac{d}{dt}(RP) \\ &= (1+\frac{\dot{R}}{c})\frac{P}{\rho}+\frac{R}{\rho c}\frac{dP}{dt} \qquad (1)\end{aligned}$$

with

$$P(R,\dot{R},t) = (P_{stat} - P_{\nu} + \frac{2\sigma}{R_0})(\frac{R_0}{R})^{3k} - \frac{2\sigma}{R} - 4\mu\frac{\dot{R}}{R} - P_{stat} + P_{\nu} - A \quad (2)$$

where $R_0 = 10\mu m$ is the equilibrium radius of the bubble, $P_{stat} = 100KPa$ is the static ambient pressure, $P_{\nu} = 2.33KPa$ is the vapor pressure, $\sigma = 0.0725$ $\frac{N}{m}$ is the surface tension, $\rho = 998$ $\frac{Kg}{m^3}$ is the liquid density, $\mu = 0.001$ $\frac{Ns}{m^3}$ is the viscosity, c=1500$\frac{m}{s}$ is the sound velocity, and k=$\frac{4}{3}$ is the polytropic exponent of the gas in the bubble. For the premier case with single frequency:

$$A = P_a \sin(2\pi\nu_1 t) \quad (3)$$

where ν_1 is the frequency of the driving sound field and P_a is the amplitude of the driving pressure.
For a dual frequency source:

$$A = P_a[\sin(2\pi\nu_1 t) + \sin(2\pi\nu_2 t + \alpha)] \quad (4)$$

where ν_2 is the secondary driving frequency and α is the phase difference.

3 Results and discussion

3.1 Bubble dynamics driven by a single frequency

The dynamics of the bubble driven by a single frequency source were studied numerically by solving equation (1). The bifurcation diagram and the lyapunov exponent spectrum of a bubble with initial radius of $10\mu m$ were sketched versus pressure and frequency in a wide range. Figures $1a - b$ show the bifurcation diagram and lyapunov spectrum of a bubble driven by $500KHz$ of frequency versus pressure within $10KPa - 2MPa$. Figure $1a$ demonstrates the highly nonlinear complex dynamics of the bubble throughout the pressure increase, which includes stable period one, period two, four, eight, chaos and then period three, period six and once more chaos. Also there are two windows of complex periodic behavior inside the second chaotic domain. Figure $1b$ shows its corresponding lyapunov exponent which confirms figure $1a$ substantiating chaotic behavior by positive values and stable periodic oscillations by negative ones.

Figures $2a-b$ show the bifurcation diagram and lyapunov spectrum of a bubble when driven by a pressure source of $500KPa$ versus frequency in the range of $100KHz - 2MHz$. We can perceive in figure $2a$ intermittent occurrence of chaotic and stable behaviors in bubble oscillations during frequency increase. The transitions from stable to chaotic oscillations are through period doubling bifurcations. The last transition to stable everlasting oscillations occurred through a saddle node bifurcation after an inverse period doubling. As a quantitative criterion for the comportment illustrated in figure $2a$, the lyapunov exponent spectrum was sketched in figure $2b$.

The significant undesirable behavior outlined in figures 1 and 2 is oscillations which are chaotic. To avoid these unwanted dynamical effects, it is necessary to carry out control strategies in the system. To achieve this objective we propose a method called "Dual frequency technique". In the stated method we introduced a secondary frequency employment along with the primary driving frequency source.

In this technique, the effect of not only the secondary frequency but also its phase difference with the basic frequency must be concomitantly taken into consideration to get the desirable outcome. More explanation is going to be put forward in this paper. In order to streamline the manifestation of the method efficacy on chaotic behaviors, some chaotic zones have been arbitrarily chosen to be exposed to the secondary frequency. For the associated zones the dynamical behavior of the bubble was analyzed before and after control. This is done through analyzing its bifurcation diagram and the corresponding lyapunov spectrum. Also time series of the normalized radius of the bubble are presented in order to reveal the stabilizing effect on the oscillations of the bubble in certain values of the control parameter.

3.2 Possibility of controlling chaotic oscillations using a dual frequency forcing

Starting with a completely chaotic zone for a single cavitation bubble when the control parameter is pressure, figure 3 has been presented. The fundamental frequency is $200KHz$ for a bubble of initial radius of $10\mu m$. Figure $3a$ displays the status prior to applying the technique; while figure $3b$ exhibits the same system when a secondary frequency of $500KHz$ was applied with the phase difference of $pi/2$ with the primer one. It is considerable in figure

$3b$ that after applying the secondary frequency, no chaotic behavior occurred within $1.55 - 2MPa$, which used to appear in the system driven by a single frequency. The according lyapunov spectra have been outlined in figure 4. The dashed line corresponds to the situation in the absence of the secondary frequency. The solid line corresponds to the secondary frequency employed situation. This figure indicates a significant abatement of the lyapunov exponent from positive values to negative ones after the proposed technique was engaged. The controlling phenomenon has also been granted by plotting the normalized bubble oscillations versus time in a certain value of the pressure before and after control in figure 5.

In order to show the capability of the proposed method to control the chaotic oscillation versus other parameters, we have plotted the bifurcation diagram of the bubble versus its initial radius, before and after control, in figures $6a - b$. The frequency and the amplitude of the driving force are $300KHz$ and $1MPa$, respectively. A secondary frequency of $1MHz$ is introduced while the phase difference is $pi/4$. Results indicate its controlling effect by reducing the chaotic oscillations to regular behaviors of period $4 \longrightarrow$ period $3 \longrightarrow$ period 8 through bubbling bifurcation and again, period 4 in the initial radius of the range $37.3\mu m$-$39\mu m$. Also the normalized oscillations of a bubble with $37.6\mu m$, before and after applying the secondary frequency, are shown in figures $7a - b$.

3.3 Effects of the phase difference on the dynamics of a dual frequency driven bubble

In bubble cavitation applications, the optimization of the control parameters like viscosity and surface tension is mainly imposed by the media. Other control parameters like pressure, initial frequency and initial radius are determined by the sort of application. In dual frequency method the secondary frequency and the phase difference can be conveniently dealt with to reach a proper choice.

Two different circumstances were considered to understand their role:

1. Under a constant secondary frequency, its phase difference with the primary frequency is varied.

2. The amplitude of the secondary frequency is varied while the phase difference is held constant.

For the purpose of studying the effect of phase difference the chaotic dynamics of a bubble with $4\mu m$ of initial radius and $1MHz$ fundamental frequency, were studied in the range of $1-3MPa$ of pressure. Figure $8a$ shows the resulting bifurcation diagram. It shows that the bubble dynamics is first chaotic, then becomes of period four regular oscillations and each of them undergoes a period doubling bifurcation and again chaotic oscillations occur in the range of $1.6-3MPa$. A secondary frequency with the value of $1.5MHz$ was applied concomitant with the fundamental. The phase differences of $pi/6$, $pi/2$ and $3pi/4$ were applied. The resulting bifurcation diagrams are presented in figures $8b-8d$ respectively. As seen in figure $8b$, by applying a phase difference of $pi/6$ a stable region appeared in the range of about $1.6-2MPa$ which were chaotic before applying the secondary frequency. When we applied the phase difference of $pi/2$ (figure$8c$) the bubble exhibited stable period four oscillations in the range of $1-1.6MPa$. As seen the oscillations are stable in the range of about $1-1.3MPa$ of pressure which they were chaotic before applying the technique. Figure $8d$ illustrates the case of applying the $3pi/4$ of phase difference. When compared with figure $8a$ we see that a stable region is formed in the range of about $2-2.5MPa$. This domain was completely chaotic prior to applying the dual frequency method. Also it should be noted that the stable region in the bifurcation diagram of the bubble driven by a single frequency source may become chaotic after applying the secondary frequency. This is obvious in figures $8b$ and $8d$.
In summary it is seen that phase difference had a distinguishable impact on the dual frequency bubble dynamics. Different stable regions appeared corresponding to typical phase differences. This suggests certain phase values can be applied to control chaotic regions of interest.

3.4 Effects of the secondary frequency on the dynamics of a dual frequency driven bubble

In order to study the impact of the value of the secondary frequency on the bubble dynamics, the chaotic oscillations of the bubble in the previous part (figure $8a$) were chosen. The phase difference was held constant with the value of $pi/3$ while the secondary frequency was $500KHz$, $800KHz$, $2MHz$ and $3MHz$. The resulting bifurcation diagrams are presented in figures $9a-d$, respectively. Comparing with figure $8a$ using a secondary frequency of $500KHz$ (figure $9a$) has stabilized the first chaotic region in the range

of about $1-1.3MPa$ to period four regular behavior. There are two small other stable regions in about $1.6-1.7MPa$ and $2.5-2.6MPa$. As seen in figure $9b$, a small domain of regular behavior around $1.2MPa$ is created when applying the value of $800KHz$ for the secondary frequency. Also there is another stable region in the range of about $1.9-2MPa$. Both of these domains were chaotic before applying the $800KHz$ of secondary frequency. Figure $9c$ shows the bifurcation diagram when the secondary frequency is $2MHz$. A vast region of stable oscillations is generated about $1.9-2.9MPa$ of pressure. The oscillations are of different periods and some bubbling bifurcations and period doublings are seen. There are other stable domains created in the range of about $1.1-1.3MPa$ and between $1.3-1.4MPa$ of pressure. Employing the secondary frequency of $3MHz$ (figure $9d$), extremely stabilizes the chaotic oscillations in the range of approximately $1.8-2.3MPa$ to period one oscillations. Also another stable region is generated about $2.6MPa$ of pressure.

In summary we see that like the influence of phase difference, the value of the secondary frequency has a strong impact on the dynamics of a dual frequency driven bubbles. Results show that by applying certain values for the secondary frequency we may be capable of controlling the chaotic oscillations in different regions of interest. Combining the suitable choice for the secondary frequency and the phase difference can help us greatly for this purpose.

3.5 Effects of the secondary frequency and phase difference on the oscillation amplitudes

As it was seen the maximum oscillation amplitudes may change when applying the dual frequency technique. In order to study the effects of variations in frequency and phase difference on the oscillations amplitude, firstly in figure $10a$ we have plotted time series of the normalized chaotic oscillations of a bubble with initial radius of $50\mu m$ driven by $300KHz$ of frequency and $1MPa$ of pressure. Then the proposed method is applied and the obtained results are presented in figure $10b-i$. In the left column the phase difference is varied for a fixed secondary frequency and in the right column the frequency is varied for a constant value of phase difference. Results demonstrate that certain values of phase difference and secondary frequency may stabilize the behavior whereas it can renovate chaos as observed in figures

10c and 10h. In figure 10 except for 10c and 10h the oscillations have become stable. It is also seen that this technique may arm us by the possibility to provide oscillations of desired amplitude.

4 Conclusion

The dynamics of an acoustically driven gas bubble was studied applying the method of chaos physics. Results indicated its rich nonlinear dynamics with respect to variations in the control parameters of the system. A method based on applying a dual frequency source is proposed. Simulation results demonstrated that the proposed procedure may be able to achieve the control objective. This is possible through choosing appropriate values of the secondary frequency and its phase difference with initial one.

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Figure1. Bifurcation diagram and the corresponding Lyapuonov spectrum of a bubble with $10\mu m$ initial radius driven with $500KHz$ of frequency versus pressure , $1a$- normalized bubble radius versus pressure, $1b$- Corresponding Lypunov spectrum.

Figure2. Bifurcation diagram and the corresponding Lyapuonov spectrum of a bubble with $10\mu m$ initial radius driven with $500KPa$ of pressure versus frequency, $2a$- normalized bubble radius versus frequency $2b$- corresponding Lyapunov spectrum.

Figure3. Bifurcation diagrams of the normalized bubble radius driven by $200KHz$ of frequency with the initial radius of $10\mu m$ versus pressure: $3a$- Chaotic behavior before applying the proposed technique, $3b$- after technique engagement with ν_2=$500KHz$ and α=$pi/2$.

Figure4. Lyapunov spectra before and after applying the proposed method. The dashed line represents the case before applying the method while the solid line represents the system after control.

Figure5. Time series of normalized bubble radius driven by $200KHz$ of frequency and $1.7MPa$ of pressure: $5a$- Chaotic oscillations, $5b$- Regular oscillations after introducing the dual frequency method (ν_2=$500KHz$ and α=$pi/2$).

Figure6. Bifurcation diagrams of the bubble normalized radius versus initial radius driven by $300KHz$ of frequency and $1MPa$ of pressure: $6a$- chaotic behavior before control, $6b$- Periodic behavior after the technique engagement (ν_2=$1MHz$ and α=$pi/4$).

Figure7. Time series of the normalized oscillations of the bubble with initial radius of $37.6\mu m$ driven by $300KHz$ of frequency and $1MP$ of applied pressure: $7a$- without applying the proposed technique, $7b$- After applying the proposed technique (ν_2=$1MHz$ and α=$pi/4$).

Figure8. Bifurcation diagrams of the normalized bubble radius driven with the fundamental frequency of $1MHz$ and R_0=$4\mu m$ versus pressure: 8a-Driven by a single frequency. $8b-d$-After applying the secondary frequency of $1.5MHz$ with the phase difference of 8b-$pi/6$, 8c-$pi/2$, 8d-3$pi/4$.

Figure9. Bifurcation diagrams of the normalized bubble radius driven by dual frequency source with fundamental frequency of $1MHz$ and phase difference of $pi/3$ and R_0=$4\mu m$ versus pressure when the secondary frequencies are: 9a-500KHz, 9b-800KHz, 9c-2MHz and 9d-3MHz.

Figure10. Normalized amplitude of oscillations versus time for a bubble with initial radius of $50\mu m$ driven by the frequency of $300KHz$ and $1MPa$ of pressure: 10a- chaotic oscillations before technique engagement, left column: after applying the secondary frequency of $\nu_2 = 1.5MHz$ while the phase difference is 10b-$pi/6$, 10c-$pi/4$, 10d-$pi/3$, 10e-pi, right column: after applying the phase difference of $pi/3$ while the secondary frequency is 10f- 400KHz, 10g- 600KHz, 10h- 1MHz and 10i-2MHZ.